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MEASUREMENT AND INTEGRATION OF ACCELERATION IN INERTIAL NAVIGATION

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ABSTRACT

Measurement and integration of acceleration in inertial navigation is associated with instrumentation problems of a character different from those in other fields of technology. The purpose of the paper is to outline the principles of such measurements, and some typical approaches to instrumentation.

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MEASUREMENT AND INTEGRATION OF ACCELERATION IN INERTIAL NAVIGATION

By J. M. Slater

INTRODUCTION

Inertial navigation (1,2,3)[1] is a relatively new art which has evolved in parallel with the development of high-performance supersonic aircraft and missiles. It is a highly specialized form of dead-reckoning. It depends upon sensing and integrating changes of vehicle motion (i.e., vehicle translational accelerations) to compute velocity, the first time integral of acceleration, and distance, the second.

Typically, the axes along which the vehicle accelerations are sensed are not the airframe axes but rather a set which is stabilized, that is to say, maintained in some predetermines space orientation which is independent of vehicle attitude and heading. Stabilization is provided by an assembly of precision gyroscopes isolated from vehicle angular movements by a set of gimbals under servo control from the gyroscopes.

Assuming such stabilization, the integrated acceleration signals can be interpreted with respect to some selected terrestrial co-ordinate system. For example, if the sensing axes are maintained level and north-south and east-west, the integral outputs can be interpreted in terms of latitude-longitude changes.

Navigation in a broad sense might be considered as a three-axis problem; determination of vehicle height, as well as latitude and longitude. For present purposes it will be considered as a two-axis problem, as it actually is on a ship. Thus, assuming the acceleration-sensing axes to be kept level or approximately so, two acceleration-sensing and measurement channels suffice.

Theoretically the complete acceleration sensing and integrating system of an inertial navigator could be a very simple device, based on a straightforward mechanization of Newton's second law of motion.

Any mass supported to be free to move along a line has the makings of an inertial double integrator, since, if an acceleration component exists along the axis, displacement is equal to the double time integral of acceleration. The acceleration sensing and integrating functions could be combined in a single element, a physical pendulum of constants such that its angular acceleration $\dot{\omega}'$ in space is equal to angular acceleration $\dot{\omega}$ of the support, in movements of the support about the center of the earth. Then angular deflection of the pendulum is equal to the angle traveled about the center of the earth -- (geocentric) latitude angle, for example, in the case of NS movement.

Fig. 1 illustrates this concept. The period of a physical pendulum $T = 2\pi(I/mgr)^{1/2}$ where I is moment of inertia and mr is the effective pendulous moment. Imagine three pendulums, all having the same mr; one has a very small I, another a quasi-infinite I and the third an intermediate I. If the pendulum support is accelerated about the center of the earth, the smallest pendulum will hang back; i. e., point behind the center of the earth. The largest one will hang forward; i. e., point ahead of the center of the earth. There must be an intermediate case where the pendu-

1 Numbers in parentheses refer to the Bibliography at the end of the paper.

lum will remain pointed at the center of the earth.

Such a hypothetical physical pendulum, with constants chosen so that $\omega' = \omega$, is the so-called Schuler-tuned pendulum. The condition for this equality is that $I = rmR$, where R is the radius of the earth. By substitution in the expression for the period given above, the period of such a pendulum, oscillating in the earth's gravitational field, is found to be $T = 2\pi(R/g)^{1/2}$, where R and g are the earth's radius and acceleration of gravity. Taking values for these quantities at the earth's surface, T = 84 min approximately. Accordingly, the hypothetical device is sometimes called the 84-min pendulum. It has been discussed at length elsewhere (1, 2, 3, 4) and for the present purposes it is sufficient to point out that direct mechanization in the simple form is impossible. For a reasonable moment of inertia I the pendulous moment mr is vanishingly small and for a convenient mr the I is colossal. Accordingly, in practical applications of the principle a convenient mr is chosen and the effect of large I is simulated by various artifices.

Practical navigation systems make use of acceleration-sensing devices which may be accelerometers in conjunction with separate integrators, or instruments in which one or two stages of integration are carried out as part of the sensing function. In all cases a feedback of the integrated output is provided to the base of the sensing device, in a manner to simulate the behavior of the simple physical pendulum.

GENERAL REQUIREMENTS OF ACCELERATION-SENSING AND INTEGRATING SYSTEMS

There may first be considered some general requirements which apply to most forms of acceleration-sensing and integrating devices for use in navigators. Such requirements are rather different from those in most other applications.

On the one hand, extremely rapid response is not required. Only those changes of vehicle motion are of interest which can produce significant changes in velocity or position. High-frequency vibratory accelerations are filtered out, when it is possible to do so, by shock mounts. Also, the environment as regards temperature, pressure, stray fields, etc., may sometimes be made more favorable than in many other air-borne applications; such variables may have to be controlled anyway for the sake of associated equipment.

On the other hand, requirements are severe as regards bias and scale factor.

The effect of an acceleration bias on navigation accuracy is relatively straightforward. In the navigator the acceleration-sensing devices constitute part of an artificial pendulum which behaves essentially like an undamped physical pendulum. Upon application, for example, of a steady horizontal acceleration bias of 0.001g the pendulous system (at 1g) will oscillate between the limits 0 and 0.002 radian -- just as would a simple pendulum upon application of such bias. The maximum position error introduced will be (0.002) R, or 7.2 nautical miles.

It is apparent that for precision navigation the bias must be kept very small. This means that bearing friction for the pendulum or equivalent acceleration responsive element must be minimized. It also means that the sensitive axis of the instrument must be maintained in an accurately predetermined direction. Shift of such axis by a small angle ϕ would allow the device to pick up an acceleration component ϕa where a is acceleration present on a quadrature axis.

The effect of accelerometer scale-factor error is similar to that of bias. If, during a 1g horizontal acceleration, the accelerometer reads 0.999g, a spurious acceleration component equal to 0.001g will appear, initiating oscillatory velocity and position errors of similar magnitude.

The effect of scale-factor error in the integral terms is not quite so simple. An error of 0.001 in the first integral would correspond to a cumulative 1 knot error at 1000 knots, were it not for the effect of g. As a matter of fact, if the first integral is "low", the tilt of the ac-

celeration-sensing device about the center of the earth will lag the corresponding vehicle movement, and a component of g will appear. This component is interpreted as an acceleration of the vehicle and accordingly the tilting drive is speeded up, in a sense to reduce the component of g. An oscillating condition, rather than a cumulative error, exists, in velocity and in position (the second integral). Without going into details, it may be assumed that the scale-factor error must be kept to the same fractional order as the bias error.

The severe requirements as regards bias and scale factor are often accompanied with a requirement that the acceleration-sensing device produce directly a signal proportional to a time integral of acceleration.

These conditions in practice bring the acceleration-sensing problem into the same order of magnitude as that of providing gyroscopes of navigation grade, which latter problem is generally recognized as being quite formidable.

There may now be considered the principles of operation of various devices intended to meet these specifications.

ACCELEROMETER WITH SEPARATE INTEGRATOR

In general, a nonintegrating accelerometer is a simpler piece of mechanism than an integrating accelerometer. For this reason accelerometers combined with separate integrators have found some use.

One of the best solutions to the problem of measuring accelerations as such was pioneered by the Germans as part of experimental V2 rocket-guidance apparatus. It consists in detecting deflection of a pendulous element with a sensitive pickoff (position transducer) and applying an electromagnetic force to drive the element toward null. Thus, the element may be provided with a coil extending into the gap of a permanent magnet, as in a d'Arsonval galvanometer movement for example. The coil is energized with dc from a servo controller taking signals from the pickoff. Restoring force is linearly proportional to coil current. The scale factor is theoretically unaffected by temperature changes except for the temperature coefficient of the permanent-magnet materials (usually about 2×10^{-4}/deg C).

Despite the inherent accuracy, simplicity and good dynamic properties of an accelerometer of this type, its applicability to navigation is limited. The difficulty lies in the step of integration. Unfortunately it is not possible to count the electrons flowing in the coil, and we are confronted with the problem of integrating a current to a high order of precision.

In certain special cases integration of accelerometer signals can be performed with sufficient accuracy by exterior means. An example is the velocity measurement scheme (5) used experimentally in some of the German V2 rockets, to signal motor cutoff on attainment of a predetermined velocity. An accelerometer was employed of the type described giving a current proportional to applied acceleration. The current was passed through an electrolytic cell, analogous to a "silver voltmeter," containing a preestablished mass of material. When all the material was decomposed, cell resistance changed abruptly and a relay was actuated. The device followed Faraday's law, $M = z \int i\,dt$, where M is mass of material decomposed, z is the electro-chemical equivalent of the material, and i is current. Such a scheme can be made simple and fairly accurate but it is not suitable for continuous indication or for feedback to base as required in true navigation systems. Also, it is difficult to carry out a double integration by this means.

In general, integration to 1-0.1 per cent is relatively simple, using motor-generator feedback tachometers or electrical circuits, for example. Mechanical integrators of ball-disk-drum type

can be made good to 0.1-0.01 per cent if adjustment of the ball position can be effected reasonably slowly, but such devices are not particularly well adapted to following rapidly changing inputs. As for electrical circuits, the necessary elements (capacitors, resistors, voltage standards, etc.) become increasingly critical as linearity and low-drift specifications are raised.

These considerations make apparent the reasons for development of integrating accelerometers.

INTEGRATING ACCELEROMETERS

It is possible to devise an acceleration-sensing device which performs a double integration by mechanization of Newton's second law of motion, without requiring impractical values of mass or radius. The trick is to arrange matters so that the moment-of-inertia member (the element which is the counterpart of the element labeled I in Fig. 1) will be free to rotate through any number of revolutions, instead of being required to rotate in strict synchronism (1:1) with vehicle angular movements about the center of the earth as in the case of the physical pendulum.

Fig. 2 will serve to illustrate the principle, in comparison with a simple nonintegrating acceleration-sensing system. At (a) is shown a typical gravity meter of bifilar-suspension type, with spring restraint. Variations in the acceleration of gravity g give rise to a combined rotation and translation of the movable mass m. For small variations in g, such as not to change the bifilar support angle α appreciably, the rotation angle ψ of the mass will be proportional to g, that is, $\psi = kg$ where k is an instrument constant depending on mass m, the geometrical configuration of the suspension, and the spring constant.

In (b) the spring restraint is removed and the restoring force for mg obtained as follows: A position pickoff detects translation-rotation of the mass, and controls, via a servo controller, a rotary servomotor which drives the suspension. Acceleration, such as g, acting on the mass member causes the member to translate and also to rotate at an angular acceleration $\ddot{\psi}$ determined solely by the moment of inertia I of the member and the configuration of the bifilar support. The motor has to rotate the support at an equal angular acceleration $\ddot{\psi}$ to keep the pickoff signal at null. Thus $\ddot{\psi} = k'g$, where k' is an instrument constant depending on the moment of inertia of the suspended mass and the geometrical configuration of the suspension. Angle ψ , the double time integral of $\ddot{\psi}$, is proportional to the double time integral of g.

The particular device sketched is not very well adapted to measuring vehicle accelerations, mainly because of its unidirectional character (it could not measure acceleration in a direction opposite to that of g). However, this disability is due to the form of suspension selected for illustration rather than to any inherent limitation. Without going into practical details, we may assume that an operative device can be built, viz., one which will sense accelerations in either direction along an axis and produce, at a motor shaft, an angular acceleration, velocity and displacement $\ddot{\psi}$, $\dot{\psi}$ and ψ proportional to acting translational acceleration and the first and second time integrals thereof.

A double-integrating accelerometer constitutes only a part of an artificial Schuler pendulum. To synthesize the required oscillatory system the integrating accelerometer must be mounted on a tiltable base, and feedback of the doubly integrated output to the base must be provided. One way of doing this is by mounting the integrating accelerometer on a trunnion, as sketched in Fig. 3, and causing the shaft to drive the trunnion through step-down gearing. Sensing is such as to tilt the trunnion in the direction of acceleration. For example, if the whole assembly is accelerated to the right the trunnion will be tipped clockwise.

The oscillatory property of the system may be demonstrated as follows:

If the appararatus is tipped through some small initial angle ϕ, so as to sense a component of

of gravity, $a = g\phi$, the rotor shaft will be accelerated, and simultaneously the trunnion will be tilted back toward level. As the device reaches level position, $a = g\phi = 0$, whereas the rotor velocity is at a maximum. The device overshoots until the rotor is stopped by a signal corresponding to $a = g\phi$. Acceleration, velocity, and displacement are each 90 deg out of phase, and the system behaves exactly like an undamped physical pendulum described by the equation

$$\ddot{\phi} + \frac{k'(g\phi)}{N} = 0$$

Wherein N is the step-down gear ratio. The period will be

$$T = 2\pi\left(\frac{N}{k'g}\right)^{1/2}$$

As mentioned previously, the requirement for Schuler tuning is that the period of oscillation be $T = 2\pi(R/g)^{1/2}$. By substitution, the required gear ratio is defined by the expression

$$N = k'R.$$

The enormous value (6×10^8 cm) of the earth-radius term R, which makes the simple form of Schuler pendulum impractical, here enters merely as a gear-reduction ratio.

An inertial navigator based on this sort of device may consist of a pair of units of the type sketched in Fig. 3, mounted relative to a gyroscopically stabilized element via angle drives in the manner described in reference (2). Compensation for earth-rotation velocity and for disturbing acceleration components must be made as in the case of any other system, whether based on an ideal Schuler pendulum or on various other artificial pendulums. When (and only when) the system has been properly tuned, and the necessary compensations made, the angular velocity and displacement of the rotor may be interpreted as proportional to vehicle velocity and position.

Double-integrating accelerometers based on a direct mechanization of Newton's second law of motion have the advantage of having virtually no inherent limitations on accuracy. Aside from the bearing for the sensing mass -- which must be very good in any type of navigation acceleration-senser -- the problem of attaining accuracy theoretically reduces itself to that of proper servo contról so that deflection of the mass is minimized, and of holding dimensions constant. Friction in the servomotor bearings does not introduce error directly and the components in the electrical channel (pickoff, servocontroller, servomotor) do not need to be either linear or highly stable in gain. Ideally, the device should be unaffected by temperature except for the very small variation in scale factor accompanying a change of dimensions of the parts.

To obtain an angular velocity signal from such a double-integrating device, when such is needed (as for compensation of Coriolis acceleration which is proportional to vehicle velocity) the output must be differentiated, as by a tachometer operated by the motor.

In some sorts of systems the quantity of primary interest is angular velocity rather than displacement. Accordingly, instruments are found which give this first integral term directly, as a shaft rotation or the equivalent.

Here again Newton's second law of motion can be invoked to perform the integration, though not in such a straightforward manner as in the case of double integration.

The gyroscope is a typical single-integrating device. Torque L (= mra where mr is a pendulous moment and a is acceleration) gives rise to precession angular velocity ρ in accordance with the relation $L = \rho H$ where H is gyroscope rotor angular momentum. By making the gyroscope un-

balanced about its output axis[2], with pendulous moment mr, ρ is made proportional to a and precession angle to the first time integral of a.

A gyroscopic pendulum integrating accelerometer was used by the Germans for measuring velocity along the trajectory of the V2 (German A4) rocket (5,6) and has recently been described in an improved form (3). The device is sketched in the upper part of Fig. 4. The gyroscope rotor is offset and is mounted in gimbals so as to have angular freedom about two axes. A pick-off detects any angular deflection of the pendulous rotor assembly about the output axis and calls up, via a servo controller, a torque on the input (outer gimbal) axis, in a sense such as to reduce the deflection.

On application of acceleration along the direction of the outer gimbal axis, precession occurs about such axis and a count of shaft revolutions gives the integral. In the V2 system scale factors were such that the gyroscope made several revolutions during the boost to cutoff velocity.

As pointed out by Clemens (1), in the case of a gyroscopically stabilized accelerometer functioning as an artificial Schuler pendulum the second time integration can be performed by the gyroscope itself, the gyroscope being torqued at a value $L = \omega H'$ where ω is angular velocity about the center of the earth and H' is angular momentum of the stabilizing gyroscope. Fig. 4 shows the single-integrating accelerometer in such a system. The gyroscope stabilizes about its input axis which is disposed at right angles to the accelerometer-sensing axis. The accelerometer applies the control torque as a function of ω, as by adjusting a precision torsional spring on the output axis of the stabilizing gyroscope. When the base is traveling at angular velocity ω about the center of the earth, the stabilizing gyroscope will have an equal space angular velocity and will thus keep the acceleration sensing axis always at right angles to R. A complete navigator inertial assembly would include two such units, disposed at right angles, and a third stabilizing gyroscope for azimuth, as described by Clemens.

As in the case of the instrument first described, the gyroscopic pendulum has no significant inherent limitations on accuracy. When practical details are considered, the complexity of the device makes realization of its potential accuracy a fairly difficult matter. The gyroscope is subject to error from a variety of sources, and the same precautions in design and use must be taken as in the case of navigation gyroscopes, though with a somewhat less degree of strictness.

In this instrument, as in any other which incorporates a gyroscope, the effect of the diurnal rotation velocity Ω of the earth and any other angular velocities of the base must be taken into account. A gyroscope is in essence a device to remain irrotational in space. Even in the absence of acceleration, the gyro-pendulum will exhibit an apparent counter-earthwise rotation at a rate equal to whatever component of Ω exists along the input axis. However, by setting scale factors so that cruise velocity corresponds to several revolutions the correction can be made noncritical.

The peculiar nature of the gyroscope also comes into play in testing the instrument. The only way in the laboratory to produce a sustained multi-g acceleration is by means of a centrifuge. But whirling the gyroscopic pendulum with the acceleration-sensitive axis disposed along a radius of the centrifuge would result in very high precession torques ($\bar{L} = \bar{\rho} \times \bar{H}$ where ρ is centrifuge spin velocity) which must be withstood by the various gyroscope bearings.

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2 In the standard nomenclature for single-axis gyroscopes (the type here under consideration) the support axis of the gyroscope gimbal is termed the <u>output axis</u> (sometimes called the "precession axis"). By construction the spin axis is established at right angles thereto. The <u>spin reference axis</u> is established by placement of the output-axis pickoff; it is the direction of the spin axis in its normal (undeflected) position. The <u>input axis</u> (sometimes called the "stabilizing axis") is defined as the axis at right angles to the output and the spin reference axes.

Other singly integrating systems based on the laws of motion are theoretically available.

An ordinary pendulum, as in a pendulum-controlled clock, can constitute an extremely accurate integrator of sustained acceleration (g). Attempts have been made to employ spring-mass systems as integrating accelerometers for use on moving bases. However, the quadratic nature of the output and the poor dynamic characteristics have militated against successful application.

Another possible device would take the form of a centrifugal governor assembly, servo-driven to maintain the flyballs at a predetermined angle. Here $\dot{\psi} = (a/r)^{1/2}$, where a is acceleration and r is radius. The quadratic nature of the response, and the mechanical complexity, are unfavorable factors.

It must not be thought that the devices mentioned exhaust the possibilities for integrating accelerometers. The problem stated in broad terms is to generate a force for constraining the movable mass to the base when acted on by an acceleration which force will be associated with, or characterized by, a time rate of change of some sort; i.e., an angular velocity, or a pulse repetition rate or other countable time series of events.

A variety of other physical phenomena besides mass reaction have the property of yielding a rate quantity proportional to force, qualitatively speaking. Many, but not all, are ruled out because the proportionality is affected by uncontrollable extraneous effects rather than being determined by some simple law.

It will be noted that the integrating accelerometers described all involve servocontrolled rotating machinery and the complications incident thereto. An ideal single-integrating accelerometer might have the form of a mass associated with a restraining transducer giving directly a frequency proportional to force exerted on it by the mass. The difficulty is in finding a transducer which will meet all the requirements of low bias, high linearity over a very wide range, high resolution, and good dynamic-response characteristics.

DESIGN CONSIDERATIONS

Some electromechanical features are common to virtually all types of acceleration-sensing devices intended for navigation use.

The sensing part of an accelerometer, whether of the nonintegrating or the integrating types, usually takes the form of a small mass, supported on a slide bearing or on a pivoted arm. The bearing in either case is designed for minimum friction or other indeterminable coercion of any sort. (In the devices sketched in Figs. 2 and 4 it is apparent that any trace of friction at the support for the sensing mass could result in malperformance.)

In earlier years, the bearing was often the weak spot in precision instruments intended for air-borne applications. Ball bearings and other supports involving Coulomb friction are of marginal utility. Quartz fibers and knife edges, so convenient for laboratory instruments, are ordinarily unsuitable for use on an accelerated base. The development of pressure-supplied gas and liquid bearings has changed the situation. Properly designed bearings of this type have strictly negligible Coulomb or static friction, and very low coercion of any sort. They also can be made largely insensitive to load changes, even those of considerable magnitude. They are rugged and resist damage in transportation and in use.

For various reasons it is usually convenient to keep the sensing mass of the accelerometer substantially fixed relative to the base, on the average, rather than to allow it to deflect any significant distance. This is strictly necessary in the pivoted-arm type, to avoid the

cross-coupling error mentioned. It is usually practically necessary even in the purely translational type (as in Fig. 2) which is immune to cross-coupling. Thus, a spring-restrained translatory mass can be a very accurate device so far as yielding a deflection proportional to acceleration is concerned. Such systems are quite practical for gravity measurement, where the total range of variation in the quantity measured is a fraction of 1 per cent. But the readout problem is formidable when a wide range of accelerations is to be covered. If it is desired to resolve one g to one part in 1000 (a relatively modest specification), even with a 0.1 in. movement the equivalent of a (frictionless) potentiometer wound with 0.1 mil wire would be necessary.

Accordingly, the pickoff will usually be employed in a null system wherein fixity of null and freedom from coercion are more important than linearity.

Variable reluctance devices such as the familiar "E" pickoff have found some use. They have the considerable advantage of requiring no lead-in wires to the sensitive element. Unfortunately, coercion is difficult to eliminate. Close control of materials and of gaps and other parts of the configuration are required to bring the stray torque level down to the required levels in navigation instruments.

Capacitor pickoff always look good in the beginning. They are simple, sensitive, and rugged. For reasons associated with their high impedance, dielectric instability, and requirements for shielding, in most development programs the capacitor pickoff sooner or later is displaced by an electromagnetic device.

Optical pickoffs, extremely useful in laboratory experimental setups, have found less application to field equipment. The large size of the optics and length of light beam to obtain the necessary high resolution militates against their use.

The variable-transformer type of pickoff in which the moving element is a coil on non-magnetic support, has every property needed. It is relatively noncritical as regards configuration or materials; it can be electrically loaded so as to have zero (or a predetermined positive or negative) coercion, and it is very sensitive. Displacement of a few microinches is readily detected. The only disadvantage is the necessity of bringing a pair of wires to the sensitive element.

The acceleration-sensing device may be provided with a force-or torque-generating transducer for application of compensating biases. Ordinarily the maximum bias will be only a small fraction of one g, if the system is one wherein the devices are kept level. Even so, linearity and repeatability tolerances are high. Furthermore, the output must be relatively independent of a slight displacement of the co-operating parts of the transducer.

The accuracy requirement usually rules out such convenient gadgets as polyphase torque motors and other a-c devices.

Devices based on attraction of iron by electromagnets obviously give the most output for a given excitation current. But without excessive refinement they are unduly subject to hysteresis and to lack of linearity.

Excellent results are obtained with moving-coil designs, analogous to the d'Arsonval galvanometer movement (as in the German accelerometer mentioned) or the driver of a loud speaker. The coil is attached to the sensitive element and is energized with dc proportional to the desired bias. Accuracy depends mainly on the constancy with which the fixed magnetic field can be maintained.

CONCLUSIONS

The variety of acceleration-sensing devices described implies that no single solution has been found to fit all inertial navigation and guidance needs. Requirements are extremely severe, and it usually pays to take advantage of any relaxation along particular lines that a given inertial system may permit.

The integrating-accelerometer problem exerts a sort of fascination on those who have to deal with it. One reason lies in the technological challenges involved in carrying out high-precision force-balancing operations in a rough environment. Another lies in the stated variety of possible lines of attack. There is only one generally useful way known to establish a self-sufficient inertial space reference; viz., by use of a gyroscope. There the integration has to be a vector integration, and this is accomplished with the aid of the law $\bar{L} = \bar{P} \times \bar{H}$. But in the case of inertial-velocity determination, while acceleration sensing has to be a vector operation, integration does not.

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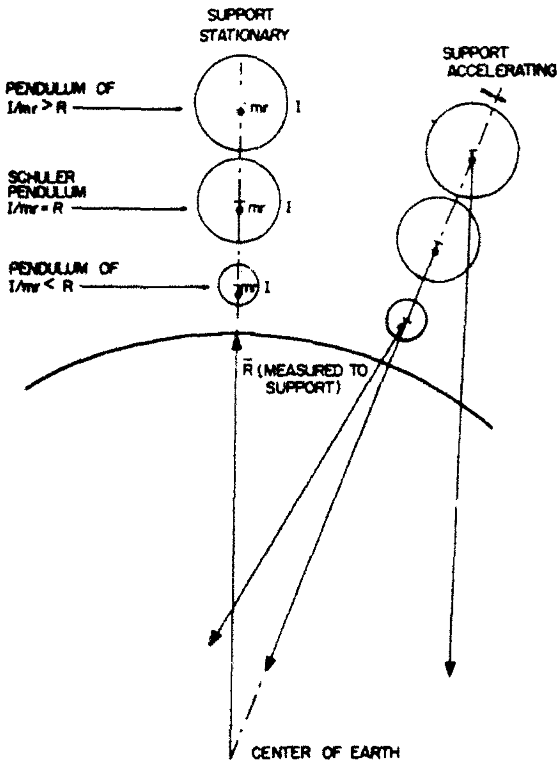

Fig.1 Behavior of pendulums of different period upon acceleration of the pivot support

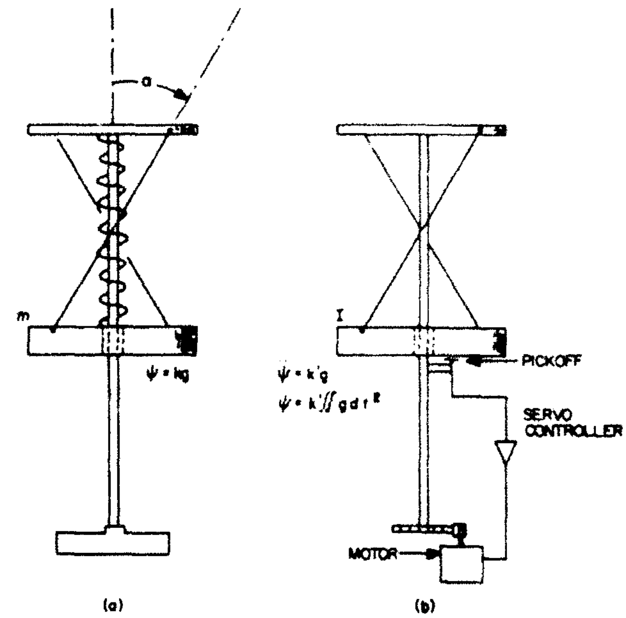

Fig.2 Nonintegrating and double-integrating g-meters

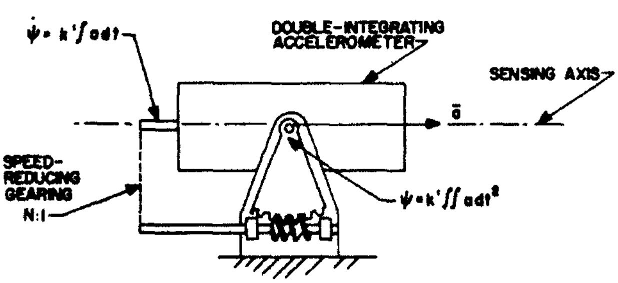

Fig.3 Double-integrating accelerometer with feedback to base, constituting an artificial long-period pendulum

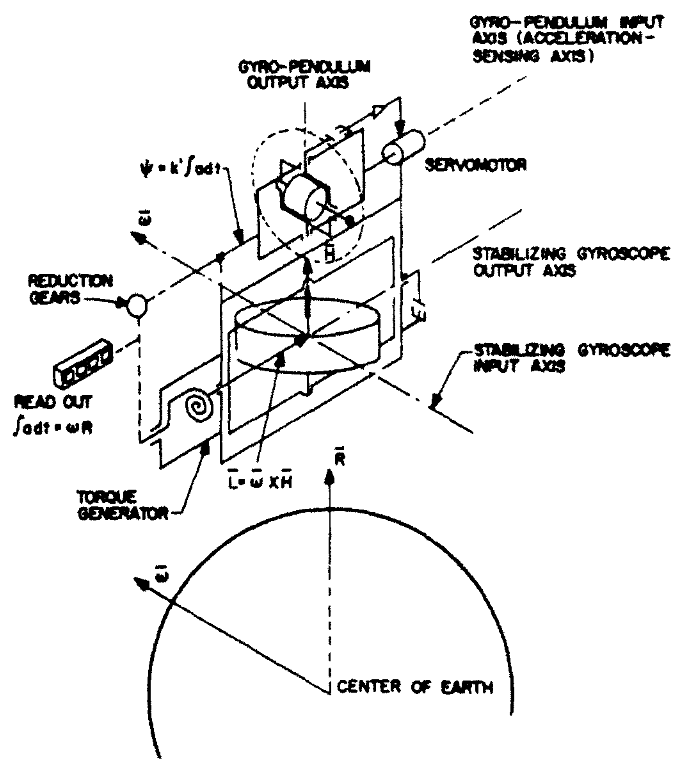

Fig.4 Single-integrating accelerometer with feedback to base, constituting an artificial long-period pendulum